obtained from existing correlations for one-dimensional flow through tube bundles. A simple assumption is used that the flow conductivity tensor depends only on the magnitude of the superficial velocity vector and *not* on its direction. This leads to some useful and interesting results, one of which is that the flow properties are isotropic for the plane perpendicular to tubes which are arranged in square and equilateral-triangle arrays. This means that the pressure drop for flow in one direction through such arrays may be predicted from data on the flow in another direction. The assumption that the flow conductivity is independent of flow direction is shown to be in reasonable agreement with existing experimental data.

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NATURAL CONVECTION FILM BOILING FROM SPHERES TO SATURATED LIQUIDS, AN INTEGRAL APPROACH

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NOMENCLATURE

A, area;
B, constant, equation (7);
C_p, specific heat at constant pressure;
D, diameter;
$F(\eta)$, velocity function, $= u/u_0$;
$G(\eta)$, temperature function $= (T-T_s)/(T_w-T_s)$;
Gr, Grashof number;
$I(\theta)$, integral, equation (8) $= \left[\int_0^\theta (\sin\theta)^{5/3}\,d\theta/(\sin\theta)^{8/3}\right]^{1/4}$;
g, acceleration of gravity;
K, thermal conductivity;
$\dot{m}$, mass flow rate, $= \rho\bar{u}\cdot 2\pi R\delta\sin\theta$;
Nu, Nusselt number;
$\overline{Nu}$, average Nusselt number, $= \frac{1}{\pi}\int_0^\pi Nu(\theta)\,d\theta$;
Pr, Prandtl number;
Pr^*, modified Prandtl number $= Pr(1+2\lambda/C_p\Delta T) = Pr(2\lambda^*/C_p\Delta T)$;
q'', heat flux;
T, temperature;
ΔT, wall superheat, $T_w - T_s$;
u, tangential component of velocity;
$\bar{u}$, average velocity, $= \frac{1}{\delta}\int_0^\delta u\,dy$;
v, radial component of velocity;
x, tangential coordinate;
y, radial coordinate.

Greek symbols

γ_1, numerical constant, $= F'(1)$;
γ_2, numerical constant, $= F'(0)$;
γ_3, numerical constant, $= G'(0)$;
γ_4, numerical constant, $= \int_0^1 F(\eta)\,d\eta$;
δ, vapor film thickness;
θ, angular coordinate;
η, dimensionless radial coordinate, $= y/\delta$;
λ, latent heat of vaporization;
λ^*, modified latent heat of vaporization, $= \lambda(1+C_p\Delta T/2\lambda)$;
ν, kinematic viscosity;
ρ, density.

Subscripts

l, liquid;
s, saturation;
w, wall.

Superscripts

$'$, differential with respect to η;
$\bar{\ }$, average value.

1. INTRODUCTION

FILM boiling heat transfer from spheres has captured more attention during the last few years particularly in the areas of cryogenics and fuel-coolant interaction in nuclear reactor safety. Attempts were made by several investigators [1–10] to understand the mechanism of natural convection film boiling from spheres to saturated liquids. Those attempts were either analytical derivations or experimental correlations. The integral method of boundary layer used in the present analysis combines both the analytical approach and experimental judgement.

2. ANALYSIS

The model presented for this analysis is shown in Fig. 1. The simplified equations of momentum and energy transport are given as follows:

An integral momentum balance over the vapour film gives:

$$-g\left(\frac{\rho_l-\rho}{\rho}\right)\sin\theta\int_0^{\delta}\mathrm{d}y = \nu\int_0^{\delta}\frac{\partial^2 u}{\partial y^2}\mathrm{d}y$$

hence:

$$-g\sin\theta\left(\frac{\rho_l-\rho}{\rho}\right)\delta = \nu\left[\left(\frac{\partial u}{\partial y}\right)_{\delta}-\left(\frac{\partial u}{\partial y}\right)_0\right]. \quad (1)$$

The energy balance is given by:

$$-K\left.\frac{\partial T}{\partial y}\right|_w \mathrm{d}A = \frac{\lambda^*}{R\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\int_0^{\delta}\rho u\sin\theta\,\mathrm{d}y\right). \quad (2)$$

The integral momentum equation (1) using non-dimensional coordinates becomes:

$$-g\sin\theta\left(\frac{\rho_l-\rho}{\rho}\right)\delta = \frac{\nu u_0}{\delta}[F'(1)-F'(0)] \quad (3)$$

hence:

$$u_0 = \frac{-g(\rho_l-\rho)\delta^2\sin\theta}{[F'(1)-F'(0)]\nu\rho}. \quad (4)$$

The integral energy equation (2) becomes:

$$-G'(0)\frac{K(T_w-T_s)}{C_p\rho\delta} = \int_0^1 F(\eta)\,\mathrm{d}\eta\,\frac{2\lambda^*}{DC_p\sin\theta}\frac{\mathrm{d}}{\mathrm{d}\theta}(u_0\delta\sin\theta) \quad (5)$$

substituting for u_0, rearranging and differentiating, we get

$$\frac{\mathrm{d}\delta^4}{\mathrm{d}\theta}-\frac{8}{3}\delta^4\cot\theta-\frac{4B}{3\sin\theta}=0 \quad (6)$$

where

$$B = \frac{\gamma_3(\gamma_1-\gamma_2)}{\gamma_4}\left(\frac{D^4}{Gr\cdot Pr^*}\right). \quad (7)$$

Equation (6) yields:

$$\delta = D\left[\frac{4\gamma_3(\gamma_1-\gamma_2)}{3\gamma_4}\right]^{1/4}\left[\frac{1}{Gr\cdot Pr^*}\right]^{1/4} I(\theta). \quad (8)$$

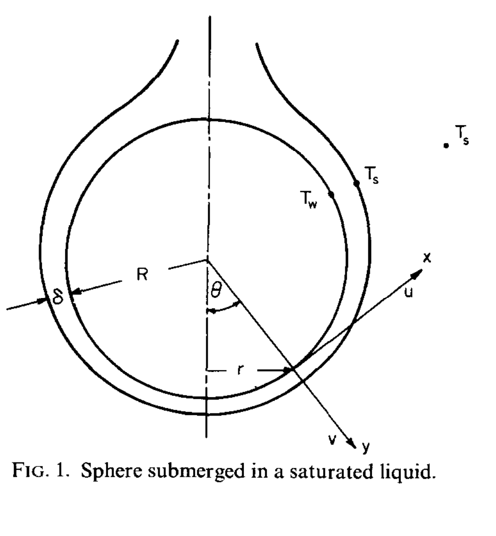

FIG. 1. Sphere submerged in a saturated liquid.

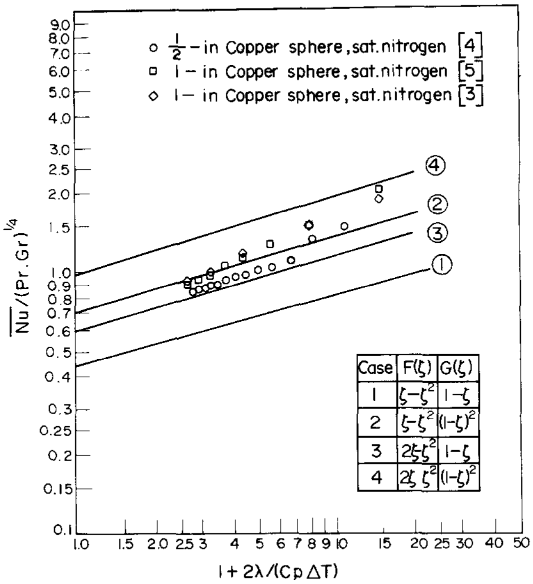

FIG. 2. The four cases considered in the analysis.

Nusselt number is given by:

$$Nu(\theta) = \frac{q''D}{K(T_w-T_s)} = -\frac{\gamma_3 D}{\delta} \quad (9)$$

hence,

$$\overline{Nu} = 0.855\left[\frac{\gamma_3^3\gamma_4}{\gamma_1-\gamma_2}\right]^{1/4}(Gr\cdot Pr^*)^{1/4} \quad (10)$$

where the integral

$$\int_0^{\pi}\frac{\mathrm{d}\theta}{I(\theta)}$$

is evaluated numerically.

Possible functional forms of $F(\eta)$ and $G(\eta)$ are assumed and γ_1, γ_2, γ_3, γ_4 are calculated. This is accomplished by considering two velocity and two temperature distributions across the vapour film. A total of four different cases are obtained.

3. COMPARISON WITH EXPERIMENTAL DATA

Only selected data from previous investigations are used here for comparison with the theoretical analysis. The criteria used are those in accordance with the assumptions of our derivation.

The experimental results of [3–5] are thus selected and plotted on Fig. 2. It is clear that case 2 closely represents the experimental results. This indicates that slip occurs at the liquid–vapour interface and that convective, in addition to conductive, heat transfer play a role in transporting heat across the vapor film. The correlating equation for case 2 is:

$$\overline{Nu} = 0.77(Gr\cdot Pr^*)^{1/4}. \quad (11)$$

In order to show the discrepancy between the above equation and the experimental data obtained at conditions not in accordance with the analysis criteria, the data of [2, 8, 10] are shown in Fig. 3. The data points of [2] representing small spheres immersed in liquid nitrogen are clearly lower than the line corresponding to equation (23). In [8], spheres are submerged in slightly subcooled water. The surface temperature is high enough for thermal radiation to be effective (400–1400°F). Subcooled sodium is the liquid used in [10], where spheres are heated to very high temperatures (2800–4200°F), before being placed in sodium. Thus in [8, 10], radiation and liquid subcooling result in the increased value of Nusselt number.

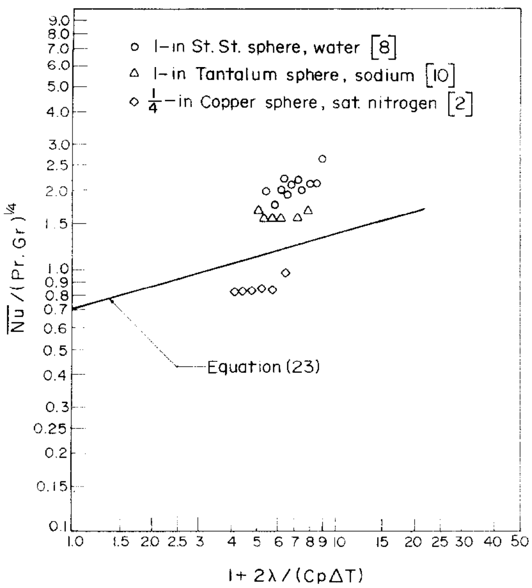

FIG. 3. Experimental points not in accordance with analysis criteria.

A survey is made of the available data points on film boiling from spheres to saturated liquids, within the framework of boundary-layer assumptions, Table 1. The points involving liquid nitrogen at atmospheric pressure, are plotted on Fig. 4, as $\overline{Nu}$ vs $(Gr \cdot Pr^*)$. The line representing equation (11) is also included. A straight line is drawn which fitted the experimental data over all ranges fairly well.

Table 1

Keys	Sphere diameter (cm)	(in)	Temperature difference between sphere and saturated nitrogen (°C)	(°F)	Reference
◇	0.95	(3/8)	38–177	(100–350)	[4]
⬙	0.64	(1/4)	38–188	(100–370)	[4]
⬘	1.27	(1/2)	21–188	(70–370)	[4]
○	2.54	(1.0)	10–204	(50–400)	[5]
▲	2.54	(1.0)	10–204	(50–400)	[3]
△	0.64	(1/4)	49–104	(120–220)	[2]
●	2.54	(1.0)	33–193	(100–380)	[6]
▽	1.27	(1/2)	>99	(>210)	[12]
▼	1.91	(3/4)	>99	(>210)	[12]

4. CONCLUSIONS

It is interesting that the analysis of the present work and that of a previous work on cylinders [11], lead to similar physical models. In both cases, it is concluded that slip occurs at the liquid–vapour interface, and that heat is transferred across the vapour film by both conduction and convection.

An attempt is made to fit the available experimental data points from several references. The best fit is found to follow a straight line that differs from equation (11), by approximately 25% at either end. The equation of that line is:

$$\overline{Nu} = 0.143(Gr \cdot Pr^*)^{0.33}. \quad (12)$$

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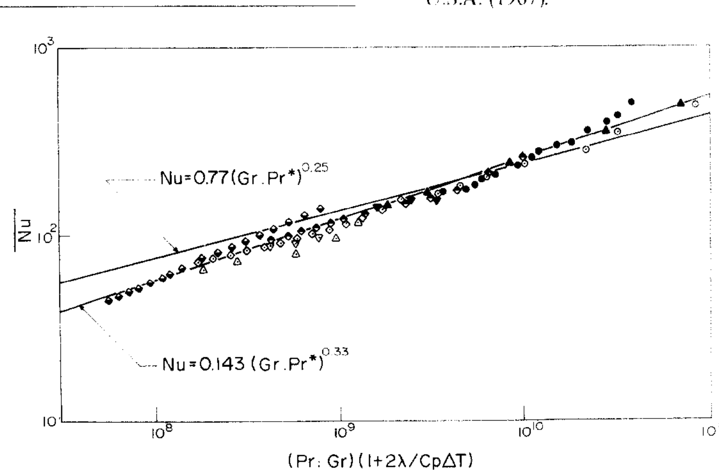

FIG. 4. Correlation of available experimental data on film boiling from spheres to saturated nitrogen at atmospheric pressure.